

The dynamics of spheres in oscillatory fluid flows.

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Abstract.

We describe a range of phenomena which occur when a collection of dense rigid spheres on a surface is subjected to horizontal oscillatory fluid flows. Pairs of equal-sized spheres are found to align perpendicular to the direction of oscillation with a well-defined gap between them. Similarly, multiple particles form chains aligned across the direction of vibration. Two unequal sized spheres are found to migrate at a constant velocity, also across the direction of vibration. These systems have been investigated both in experiment and simulation. The mechanisms responsible for these effects can be traced to the streaming flows induced by the motion of the solid spheres relative to the fluid.

Keywords: particle-laden flow, steady streaming, oscillatory flow

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Granular systems are ubiquitous in nature; the ability to control their dynamical behaviour is of great importance to many industrial processes [1]. For fluid-immersed granular systems, a number of phenomena occur as a result of the interaction between the grains and the fluid, including simple sedimentation [2], Faraday tilting [3, 4], and stripe formation in oscillatory fluid flows [5]. However, understanding the way in which rigid particles interact with each other within an oscillating fluid is still a challenging problem.

The fluid motion generated by a single, isolated, oscillating sphere has been studied both analytically and numerically [6, 7, 8, 9]. In such a system there is a non-zero, time-averaged fluid flow around the sphere called ‘steady streaming’. Recently there has been an increasing interest in the interaction between a small number of spheres induced by such streaming flows.

Wunenburger *et al.* [10] have reported experiments in which small metal spheres held in a shallow, water-filled cell were subjected to horizontal, sinusoidal vibration. The spheres formed equally spaced chains, each chain oriented perpendicular to the direction of oscillation. Voth *et al.* [11] observed pattern formation of lead spheres in a vertically vibrated cell filled with a glycerol/water mixture. A number of mechanisms have been proposed as the main source for the interactions which lead to ordering [10, 11, 12]. However, details of the interactions are still far from understood.

Here, we provide an overview of our recent experimental and computational work on the behaviour of spheres in oscillatory fluid flows. Specifically we discuss the following topics: streaming flows around a single sphere, the interaction between two identical spheres, the behaviour of a pair of unequal spheres, and chain formation in many particle systems.

METHODS USED

Experimental approach

In all our experiments we used a liquid-filled cell formed by two 3mm thick glass sheets. Each sheet was glued into an aluminium alloy frame. The two frames were separated by a butyl rubber gasket to form a liquid-tight cell 135mm long (x) and 95mm wide (y). The uniform spacing between the glass sheets could be varied but was set at 3mm thick (z) in the majority of the experiments reported here. A schematic diagram showing the orientation of the cell is shown in Fig. 1. The cell was mounted between two long-throw loudspeakers so that the plane of the glass sheets was accurately horizontal during vibratory motion and only one-dimensional motion in the horizontal plane occurred. The sinusoidal motion may be characterised by the frequency, f , and by the dimensionless acceleration of the cell, $\Gamma = A\omega^2/g$, which was determined using capacitive acceleration sensors. Here A is the amplitude of the vibration of the cell, $\omega = 2\pi f$ is the angular frequency and g is the gravitational acceleration. Typically, we use f in the range of 20Hz-70Hz and Γ from 2 to 5.

Our experiments have been conducted using glycerol/water mixtures so as to vary the kinematic viscosity. The cell was filled with the liquid and sealed so that, ideally, no air was trapped inside. The laboratory temperature was controlled to avoid changes in viscosity. In the majority of the experiments reported here the particles used were non-magnetic, stainless-steel spheres, 1mm in diameter and of density $\rho = 7950\text{kg m}^{-3}$. A high speed camera (up to 1000 f.p.s.) was mounted directly above the cell and the motion of the spheres was displayed on a monitor screen and recorded. We sometimes added marker particles [13] to the solution in order to visualise

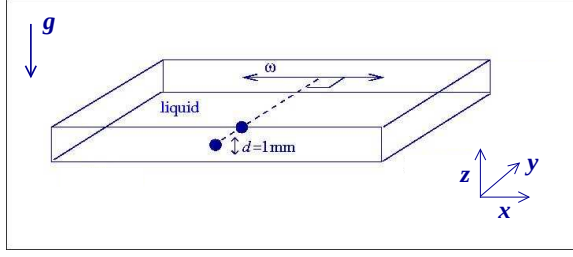


FIGURE 1. Experimental set up. The spheres rest on the lower surface of the cell. The cell is filled with liquid and is vibrated horizontally at angular frequency ω in the x -direction, as indicated by the double-headed line.

the flow patterns of the liquid.

When a cell full of liquid is subjected to vibration, the liquid, being close to incompressible, moves with the cell. The spheres move with respect to the cell because they have a greater density than the liquid. The relative amplitude of the spheres with respect to the cell, A_r , is the relevant amplitude and must be determined from experiment. Typically A_r is about $0.5A$.

Computer simulations

We have carried out numerical simulations both to aid our understanding of our experiments and to study features which are not readily accessible experimentally. The fluid and the spheres are modelled using a simplified version of the method developed by Kalthoff *et al.* [14]. The fluid is assumed to be incompressible and described by the 3D Navier-Stokes' equations. These equations are solved numerically using a finite difference approximation and the projection method [15].

The fluid-solid coupling comes from the no-slip boundary condition on the surface of the spheres: the fluid at the surface of the sphere is moving with the velocity of the sphere. This is achieved numerically by assuming that the fluid permeates the sphere and that the fluid inside the sphere moves with the instantaneous velocity of the sphere. At a given time-step, the force required to keep the fluid inside the sphere moving with the sphere's velocity is calculated from the Navier-Stokes equation. This force is then used to update the solid sphere's position using molecular dynamics techniques [16]. Because the sphere and the fluid have different densities, relative motion between the sphere and fluid is induced. Full details of the method can be found in references [17, 18, 19].

The spheres are influenced by gravity in the vertical direction, aligned perpendicular to the largest faces of the container. As a result, under horizontal vibration, the spheres move on the lower surface. However, to avoid

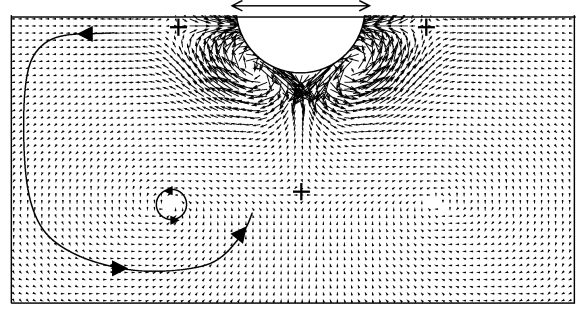


FIGURE 2. Vector plot showing the time-averaged fluid flow for a single sphere, diameter $d=1\text{mm}$, oscillating in a viscous fluid with viscosity $\nu=2 \times 10^{-6}\text{m}^2\text{s}^{-1}$. The vibratory conditions are $f=50\text{Hz}$, $A_r=0.5\text{mm}$. Arrowed lines have been drawn to indicate the direction of the outer streaming flow. Plus symbols represent stagnation points. The upper arrowed line indicates the direction of vibration.

the complications associated with lubrication forces in our simulations, the spheres are prevented from reaching the outer cells of the fluid grid by a contact force which restrains them to be at least one lattice spacing from the true cell boundary.

RESULTS

We now briefly present the results of our experiments and simulations for the four topics described in the introduction.

Single particle streaming flows

Figure 2 shows the time-averaged fluid flow around a single, oscillating sphere in an infinite liquid obtained from computer simulation. The diagram shows the flows in a plane through the centre of the sphere; only the flow on one side of the sphere is shown. The sphere oscillates along the x -axis in this plane, just below the upper, arrowed line. In this figure there are four vortices: two inner (primary streaming) and two outer (secondary streaming). In three dimensions there is rotational symmetry about the x -axis and the four vortices shown in Fig. 1 are cross-sections through four vortex rings, two inner and two outer. The circulation of each vortex ring can be seen in the figure. Note that the outer vortex rings have opposite circulation to their nearer inner vortex ring. This simulated flow pattern is in good qualitative agreement with recent experiments [20, 21].

However, in our experimental system the particles are moving in a finite box along the bottom surface. We have carried out simulations in order to determine the influ-

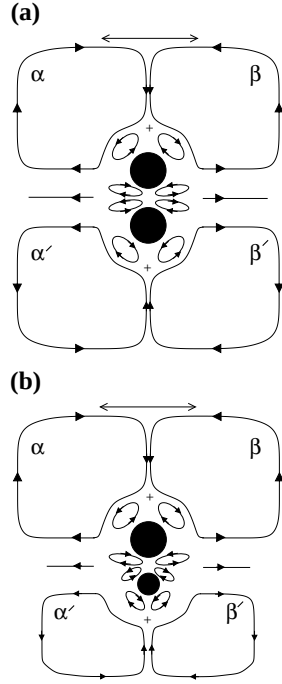


FIGURE 3. A schematic diagram showing the streaming flow in the x - y plane through the spheres for (a) a pair of equal spheres and (b) a pair of unequal spheres, moving on the lower surface of the cell. The direction of oscillation (x) is indicated by the double arrowed lines. The crosses indicate stagnation points. In each case, there are six vortex loops: four inner and two outer. The outer loops are labelled $\alpha - \alpha'$ and $\beta - \beta'$. In (a) there is rotational symmetry about the axis of oscillation whereas in (b) the symmetry is broken because the spheres are not equal in size.

ence of the confining geometry on the streaming flows. In the x - y plane through the sphere, the flow is similar to that shown in Fig. 2. However, the presence of the top and bottom surfaces breaks the rotational symmetry of the flow about the x -axis. The vortex rings (inner and outer) are somewhat squashed above the sphere and terminate on the lower surface of the cell, forming vortex loops. A similar change in the fluid flow due to the presence of a surface has recently been observed experimentally [21].

Interaction between identical spheres

When two identical spheres are oscillated in a fluid the pair attract each other and align perpendicular to the direction of oscillation [10, 11, 18]. This behaviour can be understood by considering the outer streaming flows generated by the two spheres. Widely separated spheres each follow the outer streaming flow of the other sphere until they align perpendicular to the direction

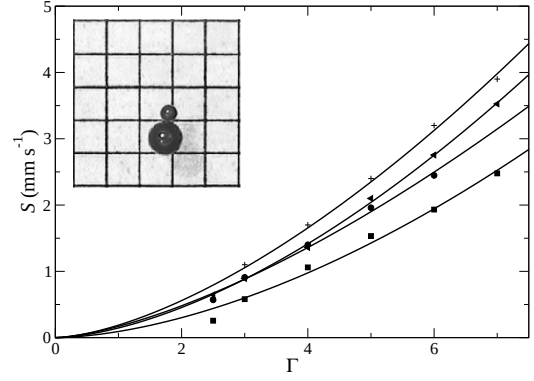


FIGURE 4. Dimer migration speed, S , as a function of Γ for a 1mm:0.5mm dimer. The frequency is 40Hz and the viscosities are $1\text{mm}^2\text{s}^{-1}$ (top), $2\text{mm}^2\text{s}^{-1}$, $4\text{mm}^2\text{s}^{-1}$ and $8\text{mm}^2\text{s}^{-1}$ (bottom). The inset shows a view from above of the dimer as it moves across the cell in the direction of the smaller sphere.

of oscillation. When the spheres approach each other, the overlapping streaming flows which have opposite circulation partially cancel, lowering the kinetic energy of the system.

Figure 3(a) shows schematically the streaming flow pattern generated by two spheres in their equilibrium configuration. Note that only two outer vortex loops remain ($\alpha - \alpha'$ and $\beta - \beta'$) and the outer streaming flow resembles that generated by a *single* particle, but with an increased magnitude. Once formed, a pair is a very stable entity.

The behaviour of a pair of unequal spheres

Figure 3(b) shows the corresponding flow pattern generated by two spheres of different diameter. It can be seen that the difference in the particle size leads to asymmetry in the streaming flow. As a result of this asymmetry, the pair is found to move in the direction of the smaller sphere.

In order to investigate this effect further, we have studied experimentally the behaviour of two, touching, stainless-steel balls held together by epoxy resin glue, as shown in the inset to Fig. 4. We report here measurements of ‘dimers’ constructed from 1mm and 0.5mm steel spheres, vibrated in fluids of different kinematic viscosity for a small range of frequencies. As soon as vibration is applied, the dimer aligns itself perpendicular to the axis of vibration and begins to move along the surface of the cell perpendicular to this axis, in the direction of the smaller sphere. Because the spheres are not neutrally buoyant, their position with respect to the cell oscillates in the vibration direction at the driving frequency. The oscillation amplitude is somewhat different for the two

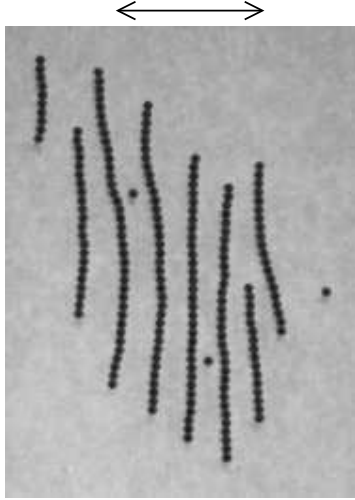


FIGURE 5. A photograph of a pattern created by a collection of 1mm stainless-steel spheres vibrated in water at 50Hz with $\Gamma = 5$.

spheres. As a result the orientation of the dimer fluctuates as it moves.

Figure 4 shows the migration speed, S , as a function of Γ for a frequency of 40Hz and a range of viscosities. The speed is found to increase rapidly with increased vibrational amplitude. We have also observed experimentally that two unattached spheres of different diameters attract each other and form a stable pair which also migrates in the same way as a glued pair.

Chain formation

Finally we consider the behaviour of many spheres subjected to oscillatory fluid flow. If the particles are initially clumped in the centre of the cell, as soon as the vibration is switched on, nearby particles align in pairs, the line joining their centres tending to orient the pair perpendicular to the direction of the oscillation. Free particles move so as to join the nearest pair and form a short chain. Short chains then evolve to form longer ones by attaching at the ends of the nearest chain. An example of a chain configuration is shown in Fig. 5.

Similar pattern formation of chains has been observed previously by Wunenburger *et al.* [10]. The existence of a well-defined spacing between chains, Fig. 5, suggests that there is both an attraction and a repulsion between chains. We have carried out simulations using the model described above to try to understand this interaction [22]. Our simulations suggest that there is only a repulsion between chains, and that chains drift apart until they fill the space available to them. However, these simulations are limited by the system size and the number of particles

so that chains become attached to the walls. One possible explanation for the stability of the chain pattern shown in Fig. 5, where the chains are not influenced by walls, is that the outer streaming flow generated by the collection of chains holds the pattern together.

CONCLUSIONS

Experiments have shown that non-neutrally buoyant spheres in oscillatory fluid flows exhibit a range of interesting phenomena. Our simulations are able to capture the essence of this behaviour and can be used to investigate the interaction mechanisms in detail. The underlying causes of these effects can be traced to streaming flow induced by the vibration.

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